An abstract geometric design on the left side of the slide. It features a diagonal line from the top-left to the bottom-right. The background is a dark blue. There are several geometric shapes and patterns: a white circle in the top-left corner; a grey semi-circle on the diagonal line; a pattern of concentric circles in the top-left; a pattern of parallel lines in the middle-left; a pattern of nested squares in the bottom-left; and various solid-colored triangles and rectangles in shades of blue, purple, and pink.

# Coup de cœur mathématique : formule de Madhava-Leibniz

Anh-Mai  
Stéphan



# PLAN :

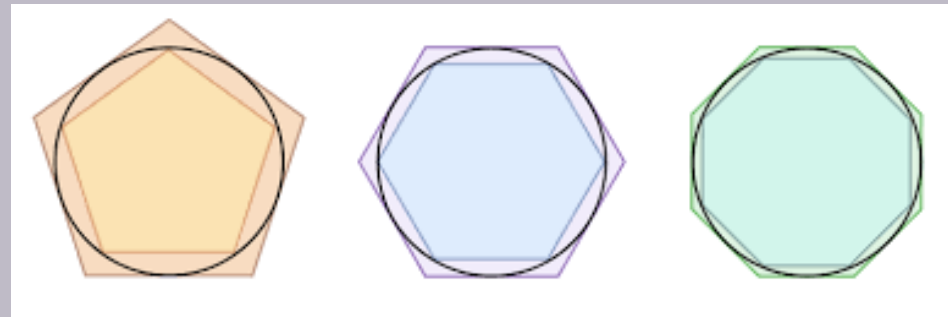
I ] Présentation brève de Pi

II ] Formule de Madhava-Leibniz

III] Conclusion sur d'autres formules

# I] PRESENTATION BRÈVE DE PI

- Déjà connu il y a 2 000 ans
- Vient du grec περιφέρεια (XVIIIème s, Euler)
- Différentes méthodes pour l'approcher (ex : Archimède)
- A partir de XV/XVIIème s: séries + calcul intégral -> plus de précision



Méthode d'Archimède pour approcher Pi

## II] FORMULE DE MADHAVA-LEIBNIZ

$$\frac{\pi}{4} = \lim(n \rightarrow +\infty) \sum_{k=0}^n \frac{(-1)^k}{2k+1}$$

➤  $G_n(x) = 1 - x^2 + x^4 - x^6 \dots - x^{4n-2}$

$$= \frac{1 - (-x^2)^{2n}}{1 - (-x^2)} = \frac{1 - x^{4n}}{1 + x^2}$$

➤  $\frac{1}{1+x^2} = \frac{1-x^{4n}}{1+x^2} + \frac{x^{4n}}{1+x^2} = G_n(x) + \frac{x^{4n}}{1+x^2}$

*On intègre entre 0 et 1 :*

$$\int_0^1 dx \frac{1}{1+x^2} = \int_0^1 dx G_n(x) + \int_0^1 dx \frac{x^{4n}}{1+x^2}$$

## II] FORMULE DE MADHAVA-LEIBNIZ

$$\int_0^1 dx \frac{1}{1+x^2} = \int_0^1 dx G_n(x) + \int_0^1 dx \frac{x^{4n}}{1+x^2}$$

Rappel :  $G_n(x) = 1 - x^2 + x^4 - x^6 \dots - x^{4n-2}$

$$[\arctan(x)]_0^1 = \left[ x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \dots \frac{x^{4n-1}}{4n-1} \right]_0^1 + \int_0^1 dx \frac{x^{4n}}{1+x^2}$$

$$\arctan(1) - \arctan(0) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots - \frac{1}{4n-1} + \int_0^1 dx \frac{x^{4n}}{1+x^2}$$

$$\text{donc } 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots \frac{-1}{4n-1} = \frac{\pi}{4} - \int_0^1 dx \frac{x^{4n}}{1+x^2}$$

$$\text{Soit : } \sum_{k=0}^{2n-1} \frac{(-1)^k}{2k+1} = \frac{\pi}{4} - \int_0^1 dx \frac{x^{4n}}{1+x^2}$$

## II] FORMULE DE MADHAVA-LEIBNIZ

$$\sum_{k=0}^{2n-1} \frac{(-1)^k}{2k+1} = \frac{\pi}{4} - \int_0^1 dx \frac{x^{4n}}{1+x^2}$$

$$\triangleright 0 \leq \int_0^1 dx \frac{x^{4n}}{1+x^2} \leq \int_0^1 dx x^{4n}$$

$$0 \leq \int_0^1 dx \frac{x^{4n}}{1+x^2} \leq \frac{1}{4n+1}$$

D'après le théorème des gendarmes :

$$\lim_{n \rightarrow +\infty} \int_0^1 dx \frac{x^{4n}}{1+x^2} = 0$$

## II] FORMULE DE MADHAVA-LEIBNIZ

$$\sum_{k=0}^{2n-1} \frac{(-1)^k}{2k+1} = \frac{\pi}{4} - \int_0^1 dx \frac{x^{4n}}{1+x^2}$$

Finalement, on a bien :

$$\lim_{n \rightarrow +\infty} \sum_{k=0}^n \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$$

- Convergence lente : 2 millions d'itérations pour 6 décimales

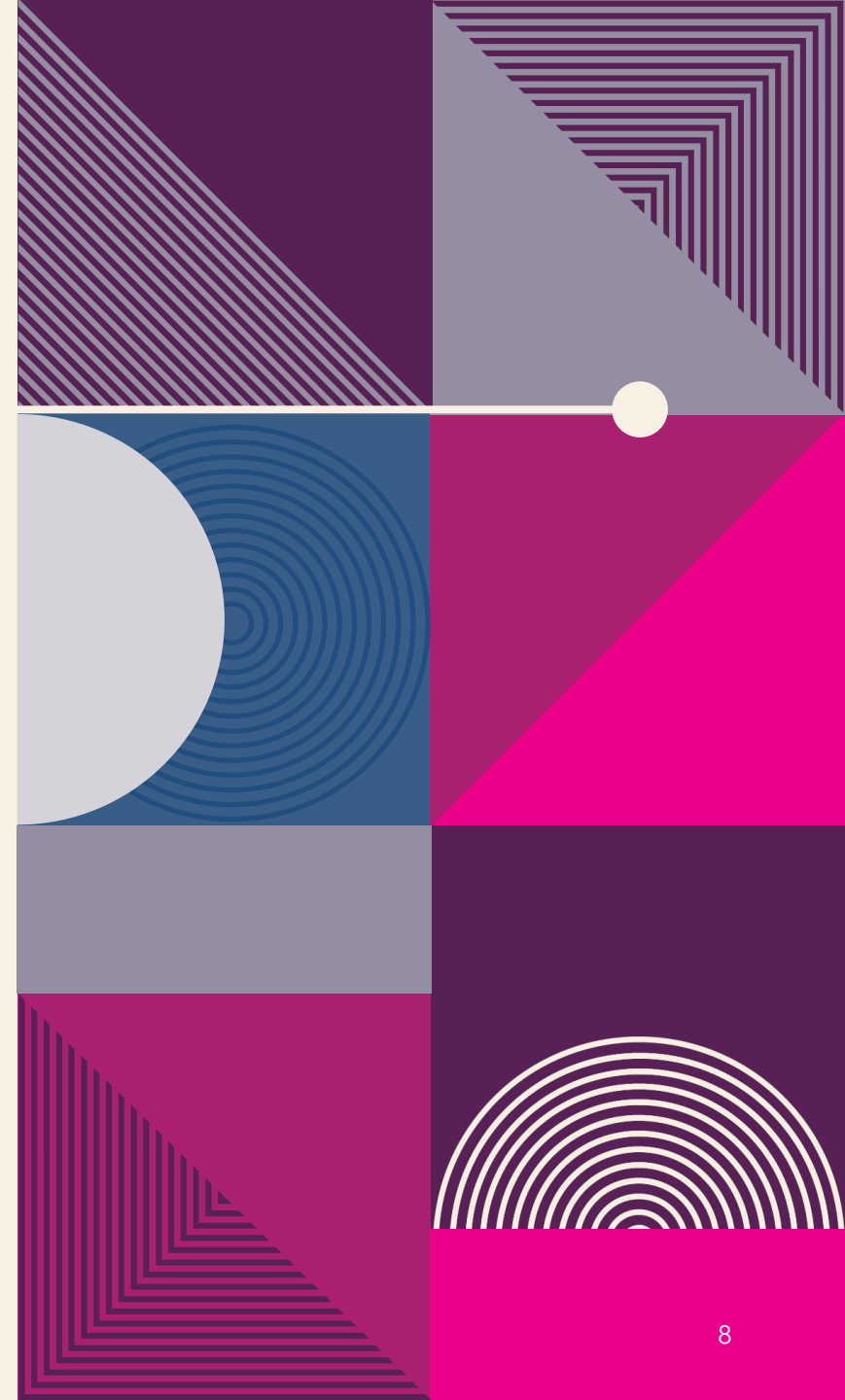
# III] D'AUTRES FORMULES

- Sommes infinies : problème de Bale (Euler)

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$$

- Produits infinis : formule de Viète

$$\frac{\pi}{2} = \frac{2}{\sqrt{2}} \times \frac{2}{\sqrt{2 + \sqrt{2}}} \times \frac{2}{\sqrt{2 + \sqrt{2 + \sqrt{2}}}} \dots$$





The background features a complex geometric design. A diagonal line runs from the top-left to the bottom-right. The top-left triangle is dark purple with a white dot and a thin white line extending from it. Below this, there are sections with concentric circles, a grey semi-circle, and pink diagonal stripes. The bottom-left corner is filled with a grid of pink and purple squares, some containing white line patterns. The rest of the background is a solid blue color.

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